Cosmology in the Randall-Sundrum Brane World Scenario

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Abstract

The cosmology of the Randall-Sundrum scenario for a positive tension brane in a 5-D Universe with localized gravity has been studied extensively recently. Here we extend it to more general situations. We consider the time-dependent situation where the two sides of the brane are different AdS/Schwarzschild spaces. We show that the expansion rate in these models during inflation could be larger than in brane worlds with compactified extra dimensions of fixed size. The enhanced expansion rate could lead to the production of density perturbations of substantially larger amplitude.

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Recently Randall and Sundrum [1,2] presented a static solution to the 5-D (classical) Einstein equations in which spacetime is flat on a 3-brane with positive tension provided that the bulk has an appropriate negative cosmological constant. Even if the fifth dimension is uncompactified, standard 4-D gravity (specifically, Newton's force law) is reproduced on the brane. In contrast to the compactified case [3], this follows because the near-brane geometry traps the massless graviton. The extension of their static solution to time-dependent solutions and their cosmological properties have been extensively studied [4–12]. In this paper, we extend further the time-dependent solution to more general situations. We shall consider only Friedman-Robertson-Walker (FRW) solutions, so the result can be expressed in terms of the Hubble constant. We shall use two approaches to study the problem. One is to solve the Einstein equation straightforwardly and obtain the Hubble constant on the brane. The bulk properties will be encoded into the behavior of the Hubble constant. We shall employ the notation and set-up of Ref [13] . The other approach starts with the solution of the bulk, and the brane is incorporated using the Israel junction condition [14,15] as first used in this context in Ref [7]. Here we consider the two sides of the brane to be described by two different Anti-deSitter(AdS), or AdS-Schwarzschild (AdSS) spaces, with different cosmological constants and different Newton constants. These cosmological scenarios, including the simplest generalizations of the original Randall-Sundrum brane world that incorporate matter on the brane [4–12], all can expand much more rapidly at early times than conventional models based only on 4-D gravity. An important consequence is that the amplitude of primordial density fluctuations produced during inflation could be substantially larger in these scenarios than in brane world models based on compactified extra dimensions of fixed size [3,16,17].

We shall consider a 5-dimensional Anti-deSitter spacetime with a positive-tension 3-brane sitting inside. We are interested in the cosmological solutions with a metric of the form

$$ds^2 = -n^2\left(\tau, y\right) d\tau^2 + a^2\left(\tau, y\right)\gamma_{ij}dx^i dx^j + b^2\left(\tau, y\right) dy^2. \tag{1}$$

The Einstein equation $G_{AB} = \kappa^2 T_{AB} = 8\pi G T_{AB}$ can be written as [10]

$$G_{00} = 3\left[\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right) - \frac{n^2}{b^2}\left(\frac{a''}{a} + \frac{a'}{a}\left(\frac{a'}{a} - \frac{b'}{b}\right)\right) + k\frac{n^2}{b^2}\right] \tag{2}$$

$$G_{ij} = \frac{a^2}{b^2}\gamma_{ij}\left[\frac{a'}{a}\left(\frac{a'}{a} + 2\frac{n'}{n}\right) - \frac{b'}{b}\left(\frac{n'}{n} + 2\frac{a'}{a}\right) + 2\frac{a''}{a} + \frac{n''}{n}\right]$$

$$+\frac{a^2}{n^2}\gamma_{ij}\left[\frac{\dot{a}}{a}\left(-\frac{\dot{a}}{a} + 2\frac{\dot{n}}{n}\right) - 2\frac{\ddot{a}}{a} + \frac{\dot{b}}{b}\left(-2\frac{\dot{a}}{a} + \frac{\dot{n}}{n}\right) - \frac{\ddot{b}}{b}\right] - k\gamma_{ij} \tag{3}$$

$$G_{05} = 3\left(\frac{n'}{n}\frac{\dot{a}}{a} + \frac{a'}{a}\frac{\dot{b}}{b} - \frac{\dot{a}'}{a}\right) \tag{4}$$

$$G_{55} = 3\left[\frac{a'}{a}\left(\frac{a'}{a} + \frac{n'}{n}\right) - \frac{b^2}{n^2}\left(\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\right) + \frac{\ddot{a}}{a}\right) - k\frac{b^2}{a^2}\right] \tag{5}$$

where γ_{ij} is a maximally symmetric 3-dimensional metric, and $k = -1, 0, 1$ parametrizes the spatial curvature. The 3-brane is placed at $y = 0$. On the two sides of the brane $(y < 0)$ and $(y > 0)$ are two different AdS spaces. The stress-energy-momentum tensor has a bulk and a brane component:

$$T_B^A = T_B^A \mid_{bulk} + T_B^A \mid_{brane} . \tag{6}$$

The $T_B^A \mid_{brane}$ component corresponds to the matter on the brane placed at $y = 0$. Since the brane is assumed to be homogeneous and isotropic, this component takes the form

$$T_B^A \mid_{brane} = \frac{\delta(y)}{b} diag\left(-\rho, p, p, p, 0\right). \tag{7}$$

The matter density on the brane is composed of a cosmological constant term, ordinary matter, radiation, etc. $T_B^A \mid_{bulk}$ is the energy-momentum tensor for the bulk matter, which consists of a non-zero cosmological constant (assumed to be different on the different sides of the brane) plus a black hole term, is

$$T_B^A \mid_{bulk} = diag\left(\Lambda_i, \Lambda_i, \Lambda_i, \Lambda_i, \Lambda_i\right) \tag{8}$$

where $i = +, -$ i.e. $i = -$ for $y < 0$ and $i = +$ for $y > 0$.

Following Ref [13], it is convenient to define $[f] = f(0_+) - f(0_-)$ to be the *jump* component for a given function f at $y = 0$, and $\{f\} = (f'(0_+) + f'(0_-))/2$ to be its *average* component at $y = 0$. The functions n, a, b, are continuous at the brane, but their derivatives are discontinuous, so the second derivatives will be of the form

$$f'' = f'' \mid_{(y \neq 0)} + [f'] \delta(y). \tag{9}$$

Using this notation, the jump part of the Einstein equation for the (00), the (ij) and the (55) components can be written as (the subscript 0 indicates that the functions are evaluated at $y = 0$)

$$\frac{2\{a'\}}{a_0 b_0} \frac{[a']}{a_0 b_0} - \frac{[a']}{(a_0 b_0)} \frac{\{b'\}}{b_0^2} - \frac{\{a'\}}{(a_0 b_0)} \frac{[b']}{b_0^2} = \frac{\kappa^2}{3} (\Lambda_+ - \Lambda_-) \tag{10}$$

$$\frac{2\{a'\}}{a_0 b_0} \frac{[a']}{a_0 b_0} + 2\left(\frac{[a']}{a_0 b_0} \frac{\{n'\}}{n_0 b_0} + \frac{\{a'\}}{a_0 b_0} \frac{[n']}{n_0 b_0}\right) - \left(\frac{[b']}{n_0 b_0} \frac{\{n'\}}{b_0^2} + \frac{\{b'\}}{b_0^2} \frac{[n']}{n_0 b_0}\right)$$

$$-2\left(\frac{[a']}{a_0 b_0} \frac{\{b'\}}{b_0^2} + \frac{\{a'\}}{a_0 b_0} \frac{[b']}{b_0^2}\right) = \kappa^2 (\Lambda_+ - \Lambda_-) \tag{11}$$

$$\frac{2\{a'\}}{a_0 b_0} \frac{[a']}{a_0 b_0} + \frac{[a']}{a_0 b_0} \frac{\{n'\}}{n_0 b_0} + \frac{\{a'\}}{a_0 b_0} \frac{[n']}{n_0 b_0} = \frac{\kappa^2}{3} (\Lambda_+ - \Lambda_-) \tag{12}$$

while the δ-function part of the Einstein equation for the (00) and the (ij) components can be written as

$$\frac{[a']}{a_0 b_0} = -\frac{\kappa^2}{3} \rho \ , \ \ \frac{[n']}{n_0 b_0} = \frac{\kappa^2}{3} (2\rho + 3p). \tag{13}$$

The remaining (the *average*) part of the (55) component of the Einstein equation is given by

$$\frac{1}{n_0^2}\left[\frac{\dot{a}_0}{a_0}\left(\frac{\dot{a}_0}{a_0} - \frac{\dot{n}_0}{n_0}\right) + \frac{\ddot{a}_0}{a_0}\right] = \frac{1}{4}\left(\frac{[a']}{a_0 b_0}\right)^2 + \left(\frac{\{a'\}}{a_0 b_0}\right)^2$$

$$+\frac{1}{4}\frac{[a']}{a_0 b_0}\frac{[n']}{n_0 b_0} + \frac{\{a'\}}{a_0 b_0}\frac{\{n'\}}{n_0 b_0} - \frac{\kappa^2}{3}\frac{\Lambda_+ + \Lambda_-}{2} - \frac{k}{a_0^2}. \tag{14}$$

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Using the above equations and switching to the proper time of the brane t defined by $d\tau = n(\tau, 0)\, dt$, the function $a_0(t) = a(t, 0)$ describes the evolution of our four-dimensional universe, and will be denoted by $R(t)$. Then the *average* component of $G_{55} = \kappa^2 T_{55}$ gives the Hubble constant $H = \dot{a}_0/a_0 = \dot{R}/R$ of the brane. The steps to follow are the same as in Ref [5,9]. After going to the proper time of the brane, the LHS of Eq(14) becomes

$$\frac{1}{R}\frac{d^2R}{dt^2} + \frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 = \frac{\dot{R}}{R}\frac{d\dot{R}}{dR} + \frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 = \frac{R}{2}\left(\frac{dH^2}{dR}\right) + 2H^2 = \frac{1}{2R^3}\frac{d}{dR}\left(H^2R^4\right). \quad (15)$$

The Hubble constant of the brane can be obtained by integrating the equation

$$\frac{1}{2R^3}\frac{d}{dR}\left(H^2R^4\right) = -\frac{\kappa^4}{36}\rho\left(\rho + 3p\right) - \frac{\kappa^2}{3}\frac{\Lambda_+ + \Lambda_-}{2} - \frac{k}{R^2} + \left(\frac{\{a'\}}{a_0 b_0}\right)^2\left[1 + \frac{3p}{\rho}\right]$$
$$+ \left(\frac{\{a'\}}{a_0 b_0}\right)\frac{(\Lambda_+ - \Lambda_-)}{\rho}. \quad (16)$$

The function $\{a'\}/a_0 b_0$ is determined by the properties of the bulk. That is, this brane function encodes informations coming from the bulk. If the two sides of the brane are identical AdS spaces, it becomes zero, as required by the Z_2 symmetry. (All functions depend on y only through $|y|$.) The other functions $\{b'\}/b_0^2$, $\{n'\}/n_0 b_0$, $[b']/n_0 b_0$, can be determined in terms of $\{a'\}/a_0 b_0$. Since $[a']/a_0 b_0$ and $[n']/n_0 b_0$ are already determined by Eq(13), then Eq (10), (11) and (12) allow us to find the three other unknown functions. We may consider expanding $\{a'\}/a_0 b_0$ in a series of even powers of $1/R$:

$$\frac{\{a'\}}{a_0 b_0} = c_0 + \frac{c_1}{R^2} + \frac{c_2}{R^4} + \frac{c_3}{R^6}\cdots \quad (17)$$

We shall see some examples where these coefficients have specific physical interpretations.

An alternative way to obtain the Hubble constant has been used in Ref [7]. The bulk is made of two pieces of five-dimensional AdSS space-time separated by the brane. The five-dimensional action is

$$S = \frac{1}{16\pi G}\int_M d^5x\sqrt{-g}\left(R + \frac{12}{l^2}\right) + \frac{1}{8\pi G}\int_{\partial M} d^4x\sqrt{-\gamma}K \quad (18)$$

The cosmological constant used here and in the Ref [7], $1/l^2$, is related to the cosmological constant Λ used in Ref [1,2,9] by $1/l^2 = \kappa^2\Lambda/6$. The brane separating the two AdSS spaces is described by the equation $r = R(t)$. The extrinsic curvature of the brane, $K^{ab} = \nabla^a n^b$ can be written as $K_{\mu\nu} = n^c\partial_c\gamma_{\mu\nu}/2$ where $\gamma_{\mu\nu}$ is the induced metric on the brane, and n^a is the unit normal of the brane. The indices a, b cover the 5D space-time, the indices μ, ν cover the 4D space-time, and the indices i, j cover the space coordinates of the brane. The junction condition at the brane as required by the Einstein equations is

$$\frac{K_{ij}^+}{G_+} - \frac{K_{ij}^-}{G_-} = -8\pi\left(T_{ij} - \frac{1}{3}T\gamma_{ij}\right) \quad (19)$$

where $T_{\mu\nu}$ is the energy-momentum tensor of matter on the brane, and T is its trace. Here we generalize Eq(18) to allow the possibility that the Newton's constants on the two sides are

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different. This scenario can happen in string theory, when both the cosmological constant and the Newton's constant of an AdS space are related to the number of D-branes present. So we may visualize the situation where the two AdS bulk spaces separated by the brane may have different Newton's constants and cosmological constants. We may also entertain the possibility that the cosmological constants on the two sides are different, a situation that can arise when the brane is a thin-wall approximation of a domain wall [18]. Note that the solution of the jump condition yields a solution to the 5-D Einstein equation, in contrast to the case where the brane is treated as a probe [22].

If the bulk metric has the form

$$ds^2 = -f(r) dt^2 + r^2 d\Sigma_k^2 + f^{-1}(r) dr^2 \tag{20}$$

then the velocity vector of the brane u^μ, which satisfies $u^\mu u_\mu = -1$ and $n^\mu u_\mu = 0$ is given by $u^t = \left(f + \dot{R}^2\right)^{\frac{1}{2}} f^{-1}$ and $u^r = \dot{R}$, where $\dot{R}$ being the derivative with respect to the proper time τ. Up to a sign, the unit normal to the brane is given by $n^t = -f^{-1}\dot{R}$ and $n^r = -\left(f + \dot{R}^2\right)^{\frac{1}{2}}$. The minus sign is due to the fact that the coordinate r is decreasing in the direction n^μ. With these components, the spatial components of the extrinsic curvature on the two sides of the brane are

$$K_{ij}^- = \frac{\left(f_- + \dot{R}^2\right)^{\frac{1}{2}}}{R}\gamma_{ij} \ , \ K_{ij}^+ = -\frac{\left(f_+ + \dot{R}^2\right)^{\frac{1}{2}}}{R}\gamma_{ij} \tag{21}$$

where the relative signs of K_{ij}^- and K_{ij}^+ follow from the definition of the unit normal n^μ.

For a matter tensor on the brane of the form given in Eq(7), $T_{ij} - \frac{1}{3}T\gamma_{ij} = (\sigma + \rho_m)/3$, and the equation describing the evolution of the brane (3-brane) becomes

$$\frac{\left(f_- + \dot{R}^2\right)^{\frac{1}{2}}}{G_-} + \frac{\left(f_+ + \dot{R}^2\right)^{\frac{1}{2}}}{G_+} = \frac{8\pi(\sigma + \rho_m)}{3}R. \tag{22}$$

Using the above equation and the notation $\tilde{\lambda} = 8\pi(\sigma + \rho_m)/3$, the Hubble constant is found to be:

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{\tilde{\lambda}^2\left(G_+^2 + G_-^2\right)G_+^2 G_-^2}{\left(G_+^2 - G_-^2\right)^2} + \frac{\left(f_+ G_-^2 - f_- G_+^2\right)}{R^2\left(G_+^2 - G_-^2\right)}$$

$$-\frac{2G_+^2 G_-^2}{\left(G_+^2 - G_-^2\right)^2}\left[\tilde{\lambda}^4 G_+^2 G_-^2 + \frac{\tilde{\lambda}^2}{R^2}(f_+ - f_-)\left(G_+^2 - G_-^2\right)\right]^{\frac{1}{2}} \tag{23}$$

For an explicit example, we can consider the metric given in Ref [19],

$$f_\pm = k + \frac{R^2}{l_\pm^2} - \frac{\mu_\pm}{R^2}. \tag{24}$$

This reduces to the case considered in Ref [7] if we set $G_+ = G_- = G$ and $l_+ = l_- = l$. The existence of a horizon at $r_{h_i}^2 = \frac{l_i^2}{2}\left(-k \pm \sqrt{k^2 + 4\mu_i/l_i^2}\right)$ imposes restrictions on the values of

μ_i depending on the value of k. For $k = +1$, a positive value of $r_{h_i}^2$ imposes $\mu_i > 0$, while for $k = -1$, the condition is $\mu_i > -l_i^2/4$.

We may also consider a more general metric. Motivated by the generalized AdSS solution of Ref [20], we can consider a metric of the form

$$ds^2 = -\omega_i^{-2} f_i dt^2 + \omega_i \left(f_i^{-1} dr^2 + r^2 d\Omega_{3,k} \right) \tag{25}$$

where $f = k - \frac{\mu}{r^2} + \frac{r^2}{l^2}\omega^3$. In Ref [20], $\omega^3 = H_1 H_2 H_3$ with $H_I = 1 + \frac{q_I}{r^2}$, where q_I are charges. For $\omega^3 = 1$ and $G_+ = G_-$, it reduces to the metric of Ref [19]. In general ω is a function of r. In this case, we have

$$u^r = \dot{R}, \ u^t = \left(\dot{R}^2 + \frac{f}{\omega} \right)^{1/2} \frac{\omega^3}{f}, \ n^t = -\frac{\omega^{3/2}}{f}\dot{R}, \ n^r = -\left(\frac{f}{\omega} + \dot{R}^2 \right)^{1/2}. \tag{26}$$

It is now straightforward to use the jump condition to obtain an expression for the Hubble constant. To match the two approaches we have discussed, let us go back to the simple case where $\omega = 1$ and $G_+ = G_-$. The equality $G_+ = G_- = G$ allows us to define $\lambda = G\tilde{\lambda}$. We can solve for $\dot{R}$ and obtain the Hubble constant:

$$H^2 = \left(\frac{\dot{R}}{R} \right)^2 = \frac{\lambda^2}{4} - \frac{f_- + f_+}{2R^2} + \frac{(f_- - f_+)^2}{4R^4\lambda^2} \tag{27}$$

Using the expressions for the functions f_i, the Hubble constant becomes

$$H^2 = \frac{\lambda^2}{4} - \frac{1}{2}\left(\frac{1}{l_-^2} + \frac{1}{l_+^2} \right) + \frac{1}{4\lambda^2}\left(\frac{1}{l_-^2} - \frac{1}{l_+^2} \right)^2 + \frac{k}{R^2}$$

$$+ \frac{1}{R^4}\left\{ \frac{\mu_- + \mu_+}{2} - \frac{\mu_+ - \mu_-}{2\lambda^2}\left(\frac{1}{l_-^2} - \frac{1}{l_+^2} \right) \right\} + \frac{1}{R^8}\frac{(\mu_+ - \mu_-)^2}{4\lambda^2}. \tag{28}$$

To compare Eq(16) with this expression, we must first integrate Eq(16), which will generate an integration constant. We see that the integration constant is

$$C = \frac{\mu_- + \mu_+}{2} - \frac{\mu_+ - \mu_-}{2\lambda^2}\left(\frac{1}{l_-^2} - \frac{1}{l_+^2} \right) \tag{29}$$

where we identify

$$\lambda = \frac{\kappa^2 \sigma}{3}, \ \frac{1}{l_i^2} = \frac{\kappa^2 \Lambda_i}{6} \tag{30}$$

and $c_0 = -\kappa^2 (\Lambda_+ - \Lambda_-)/12$, $c_1 = 0$, $c_2 = -(\mu_+ - \mu_-)/2\lambda$. The integration constant was obtained in Ref [7,9,10], and the R^{-8} term was first obtained in Ref [7]. The μ term has been interpreted as due to a $N = 4$ super-Yang-Mills theory on the brane via the AdS/CFT correspondence [12]. The holographic principle [21] states that information in the bulk is encoded in the data on the boundary. Intuitively, one may understand this as a boundary value problem. Evolution of the boundary to the bulk fixes the properties of the bulk in terms of the values at the boundary. However, this intuition does not apply to the brane world

scenario when there are different AdSS spaces on the two sides of the brane. It is natural to ask if data on the brane encodes all properties of both bulks, or only a combination of the bulk information with no chance to dis-entangle them. The presence of the $(\mu_+ - \mu_-)^2/R^8$ term in addition to the $(\mu_+ + \mu_-)^2/R^4$ term in the Hubble constant equation allows one to determine both μ_+ and μ_-. That is, there are two different conformal field theories on the brane, which couple to each other. This suggests that there is enough information on the brane to determine fully all the bulk properties on each side of the brane. In this sense, the holographic principle is deep.

Tuning the effective cosmological constant in the brane to zero requires

$$\frac{\lambda^2}{4} - \frac{1}{2}\left(\frac{1}{l_-^2} + \frac{1}{l_+^2}\right) + \frac{1}{4\lambda^2}\left(\frac{1}{l_-^2} - \frac{1}{l_+^2}\right)^2 = 0 \tag{31}$$

or using $\lambda = \frac{\kappa^2\sigma}{3}$,

$$\frac{\kappa^4\sigma^2}{36} - \frac{\kappa^2\left(\Lambda_- + \Lambda_+\right)}{12} + \frac{\left(\Lambda_- - \Lambda_+\right)^2}{16\sigma^2} = 0. \tag{32}$$

For $\Lambda_- = \Lambda_+$, this reduces to the Randall-Sundrum case. Otherwise, the equation has the two solutions

$$\kappa^2\sigma^2 = \frac{3}{2}\left(\sqrt{\Lambda_-} \pm \sqrt{\Lambda_+}\right)^2. \tag{33}$$

the $''+''$ solution being the one found in Ref [23]. Note that, if we choose either Λ to be zero, the two solutions for the value of the brane tension merge, and the minimum of H^2 is exactly zero. By choosing $\mu_i = 0$ and $\rho_m = 0$ in Eq(28) we obtain

$$H^2 = \frac{\kappa^4\sigma^2}{36} - \frac{\kappa^2\Lambda}{12} + \frac{\Lambda^2}{16\sigma^2} = \left(\frac{\kappa^2\sigma}{6} - \frac{\Lambda}{4\sigma}\right)^2 \tag{34}$$

This intriguing property implies that the minimum of the 4-dimensional effective cosmological constant is bounded below by zero.

The relationship between the 5D and 4D Newton constants is

$$\frac{l_+}{G_+} + \frac{l_-}{G_-} = \frac{2}{G_{(4)}}. \tag{35}$$

This can be obtained by dimensional reduction. We may also identify the 4D Newton constant by adding matter to the brane, expanding for $\rho_m \ll \sigma$ and identifying the coefficient of ρ_m in Eq(28).

This shows that conventional cosmology can be recovered at large enough values of R, where the matter density is low and the additional terms in Eq(28) become small. Choosing the "$-$" solution of Eq(33) does not, however, lead to conventional cosmology at large enough R.

It is instructive to consider a simplified version of Eq(28) to better appreciate its significance. To be concrete, let us assume that $k = 0 = \mu_+ - \mu_-$, but still allow $l_- \neq l_+$. Then we find that

$$H^2 = \frac{(\lambda^2 - \lambda_+^2)(\lambda^2 - \lambda_-^2)}{4\lambda^2} + \frac{\mu}{R^4}, \tag{36}$$

where $\mu = \mu_\pm$ and $\lambda_\pm^2 = (1/l_+ \pm 1/l_-)^2$; quite generally, this is to be augmented by an equation governing the evolution of λ, which can always be written schematically as $\dot{\lambda} = -3H(1+w)\lambda$, where for mixtures of matter, radiation, and evolving fields, $-1 \leq w \leq 1$, so λ decreases with expansion generically. It is easy to see that the first term in Eq(36) is negative for any $\lambda^2 < \lambda_+^2$, which means that for such values of λ, there are only sensible cosmological models if $\mu > 0$. In such a case, one expects that the Universe expands to a maximum R and then recollapses, generically, although it is conceivable that the parameters could be fine-tuned to avoid recollapse and attain $\lambda \to \lambda_-$ as $R \to \infty$. However, it is clear that the μ/R^4 term plays an essential role in the expansion rate throughout the history of such models, whether they collapse or expand forever, and consequently conventional cosmology is not recovered in any limit. Thus, such models could never reproduce the successes of cosmological nucleosynthesis theory, for example, and would not yield acceptable theories of large scale structure, even if expansion to very large (or infinite) R is possible.

On the other hand, for $\lambda > \lambda_+$, the Universe always expands, irrespective of the sign of μ, although, for $\mu < 0$ it expands from a minimum value of R. As long as there is some component in λ with $w < 1/3$, such as the brane tension or the vacuum energy density of a brane field, the μ/R^4 tern in Eq(36) becomes progressively less important as the Universe expands, and conventional cosmology can be recovered provided that $\lambda \to \lambda_+$ as $R \to \infty$.

During the inflation era, the matter density, ρ_m, is dominated by the inflaton, which we will take to be a single component scalar field ϕ with some effective potential $V(\phi)$. The inflaton will tend to roll toward its potential minimum, and, while the field is rolling slowly, the energy density in the field is dominated by $V(\phi)$, which decreases with time as the Universe expands, but only slowly if $V(\phi)$ is flat enough. If the inflation commences with $\lambda > 1/l_+ + 1/l_-$, then H^2 decreases as ϕ rolls toward the minimum of $V(\phi)$, where $V(\phi) = V_{min}$. Provided that $V_{min} + \sigma$ is tuned so that $\kappa^2(V_{min} + \sigma)/3 = \lambda_+$, conventional cosmology can be recovered. [1]

One of the intriguing features of Eq(28) is that for λ well above $1/l_+^2 + 1/l_-^2$, the expansion rate grows linearly with λ, i.e. $H \approx \lambda/2$ (assuming that the other terms in Eq(28) that are proportional to inverse powers of R^2 can be neglected).[2] With $\rho_m \approx V(\phi)$, this limit applies when $V(\phi)$ is at least a factor of a few larger than $\sigma_+ = \sqrt{3/2}(\sqrt{\Lambda_+} + \sqrt{\Lambda_-})/\kappa$. The expansion rate during this epoch, $H \approx \kappa^2 V(\phi)/6$, can be much larger than the expansion rate in conventional inflation for the same value of $V(\phi)$, which is $H_{conv} = (8\pi V(\phi)/3M_P^2)^{1/2}$, where M_P is the Planck mass: $H/H_{conv} \approx \kappa^2 M_P \sqrt{V(\phi)/96\pi} = \sqrt{V(\phi)/8\sigma_+}(\sqrt{l_+/l_-} + \sqrt{l_-/l_+})$. Conceivably, $H \to H_{conv}$ only as ϕ settles into its potential minimum, and during much

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[1]Note that to the extent that we may think of σ as the tension of a domain wall solution for self-gravitating supergravity fields off the brane, and V_{min} as arising from standard model fields that are confined to the brane, the precise cancellation of the cosmological constant involves cooperation between brane and bulk physical fields.

[2]This is also true in scenarios where the AdS spaces on either side of the brane are identical.

of inflation $H \approx \kappa^2 V(\phi)/6$ instead. This can happen in a number of ways. First, if $V(\phi)$ has some characteristic scale V_0, which can occur if the effective potential is flat until it plummets to its minimum at $V_{min} \ll V_0$ (or $V_{min} = 0$), and $V_0 \gg \sigma_+$, then $H \approx \kappa^2 V_0/6$ for most of the inflationary era. Second, if $V(\phi)$ is an increasing, polynomial function of ϕ, for example a simple powerlaw $V(\phi) \sim \phi^n$, then $H = \kappa^2 V(\phi)/6$ at sufficiently large values of ϕ, and the conventional expansion rate is only achieved when $V(\phi) \lesssim \sigma_+$. Finally, when either l_+ or l_- is much larger than the other, then the expansion rate can be $H \approx \kappa^2 V(\phi)/6$ even when $V(\phi)$ is only a factor of a few larger than σ_+.

Generally speaking, this enhanced expansion rate would have no consequence for the mean properties of the Universe; reheating and the end of inflation would be unaffected. But, the production of density perturbations from fluctuations in the inflaton field could be dramatically different than in models for brane world inflation with compactified extra dimensions of fixed size, where $H = H_{conv}$ and $\delta\rho/\rho$ generally is too small [3,16,17]. To evaluate the differences, suppose that $\sigma = \sigma_+$ and $V_{min} = 0$. We estimate the magnitude of the density fluctuations produced from the usual relationship $(\delta\rho/\rho)_q \sim (V'(\phi)H/\dot{\phi}^2)_{h.c.} \sim (H^3/V')_{h.c.}$, where $V' = \partial V(\phi)/\partial\phi$, and the subscript "h.c." means that we evaluate H^3/V' when a perturbation of comoving scale q^{-1} crosses the horizon during inflation [25]. Two simple examples will suffice to illustrate the effect. If $V(\phi) = m^2\phi^2/2$, an example of "chaotic inflation" (e.g. [24]), then we estimate $\delta\rho/\rho \sim \kappa m^{3/2} N_q^{5/4}$, which is to be compared to the value $(\delta\rho/\rho)_{conv} \sim (m/M_P)N_q$ for $H = H_{conv}$, where N_q is the number of e-foldings of $R(t)$ remaining in inflation after horizon crossing for a comoving scale q^{-1}. The fluctuations on scales that cross the horizon while $H \approx \kappa^2 V(\phi)/6$ differ from what one would get for $H = H_{conv}$ by a factor $\sim \kappa m^{1/2} M_P N_q^{1/4}$. If instead $V(\phi) = V_0[1 - \exp(-\phi/m)]$, which arises in "brane inflation" (e.g. [16], [17]), then we estimate $\delta\rho/\rho \sim \kappa^2 V_0 N_q/m$, which differs from the estimate $(\delta\rho/\rho)_{conv} \sim V_0^{1/2} N_q/M_P m$ found for $H = H_{conv}$ by a factor $\sim \kappa^2 V_0^{1/2} M_P$. If the characteristic energy scales of these inflationary models are comparable to $\kappa^{-2/3}$, and $M_P \gg \kappa^{-2/3}$, then the implied density fluctuations are much larger than for $H = H_{conv}$ with the same inflaton potentials. The enhancement in $\delta\rho/\rho$ for these two models is a consequence of the faster expansion rate during inflation, so that the H^3 factor in our estimate of $\delta\rho/\rho$ is larger than H_{conv}^3, but is mitigated by the evolution of ϕ, which tends to raise the value of V' at horizon crossing for these potentials. [3] A similar effect is seen in brane world cosmologies in which the extra dimensions are compactified, provided that the extra dimensions are smaller, and therefore the effective Newton "constant" is larger, during inflation than today [17,26].

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[3]There are also potentials for which there is no effect, and for which the suppression due to V' outweighs the enhancement due to H^3. Thus, for $V(\phi) = \lambda\phi^4/4$ or $V(\phi) = V_0 - \lambda\phi^4/4$. we estimate $\delta\rho/\rho \sim \lambda^{1/2} N_q^{3/2}$, just as in conventional inflation, and for $V(\phi) = m^{4-n}\phi^n/n$ with $n > 4$ the density fluctuations may even be smaller than for $H = H_{conv}$.

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